

# GATE QUESTIONS

Dronacharya College of Engineering, Gurgaon

Subject : Signals and System

Branch ECE/EEE

Year 2021

Q1. The Exponential Fourier Series representation of a continuous -Time periodic Signal  $x(t)$  is define as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Were  $\omega_0$  is the fundamental angular frequency of  $x(t)$  and the Coefficients of the series are  $a_k$ .The following information is given about  $x(t)$  and  $a_k$ .

- i.  $X(t)$  is real and even ,having a fundamental period of 6
- ii. The average value of  $x(t)$  is 2
- iii.  $a_k = \begin{cases} k & 1 \leq k \leq 3 \\ 0, & k > 3 \end{cases}$

The average power of the signal  $x(t)$  (rounded off to one decimal place)

Q2. For a unit step input  $u[n]$ , a discrete -time LTI system produces an output signal  $(2\delta[n + 1] + \delta[n - 1])$ . Let  $y[n]$  be the output of the system for an input  $((1/2)^n + u[n])$ . The value of  $y[0]$  is

Q3. Consider the signals  $x[n] = 2^{(n-1)} u[-n+2]$  and  $y[n] = 2^{(-n+2)} u[n+1]$  Where  $u[n]$  is the unit step sequence. Let  $X(e^{j\omega})$  and  $Y(e^{j\omega})$  be the discrete time fourier transform of  $x[n]$  and  $y[n]$ , respectively. The value of the integral

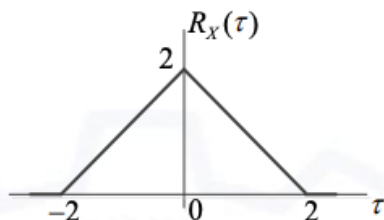
$$\frac{1}{2\pi} \int_0^{2\pi} (e^{j\omega}) Y(e^{j\omega}) d\omega$$

Rounded off to one decimal Place is

Q4. Consider two 16-point sequences  $[n]$  and  $h[n]$ . Let the linear convolution of  $[n]$  and  $h[n]$  be denoted by  $y[n]$ , while  $z[n]$  denotes the 16-point inverse discrete Fourier transform (IDFT) of the product of the 16-point DFTs of  $x[n]$  and  $h[n]$ . The value(s) of  $k$  for which  $[k] = y[k]$  is/are

- A )  $k=0,1,2, \dots, 15$  B)  $k=0$                       C)  $k=15$                       D)  $k=0$  and 15

Q5 The autocorrelation function  $R(\tau)$  of a wide-sense stationary random process  $X(t)$  is shown in the figure.



Q6. If the input  $x(t)$  and output  $y(t)$  of a system are related as  $y(t) = \max(0, x(t))$ , the the system is  
 A) linear and Time Variant B) linear and Time Invariant C) Non- linear and Time Variant D) Non linear and Time – Invariant.

Q7 Two discrete-time linear time invariant system with impulse response

$h_1[n] = \delta[n - 1] + \delta[n + 1]$  and  $h_2[n] = \delta[n] + \delta[n - 1]$  are connected in cascade, where  $\delta[n]$  is the Kronecker delta .The impulse response of the cascaded system is



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D) only  $X[3]$  and  $X[5]$  are non-zero

Q15. Consider a Linear time-invariant system whose input  $r(t)$  and output  $y(t)$  are related by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 4y(t) = 6r(t)$$

The poles of this system are at

a)  $+2j, -2j$  b)  $+2, -2$  c)  $+4, -4$  d)  $+4j, -4j$

Q16. Which of the following option is true for a linear time-invariant discrete time system that obeys the difference equation:

$$y[n] - ay[n-1] = b_0x[n] - b_1x[n-1]$$

- a)  $y[n]$  is unaffected by the value of  $x[n-k]$ ;  $K > 2$ .
- b) the system is necessarily causal
- c) the system impulse response is non-zero at infinitely many instants
- d) when  $x[n]=0, n < 0$ , the function  $y[n]; n > 0$  is solely determined by the function  $x[n]$

Q17. Consider a signal  $x[n] = \left(\frac{1}{2}\right)^n 1[n]$ , where  $1[n] = 0$  if  $n < 0$ , and  $1[n] = 1$  if  $n \geq 0$

The Z-transform of  $x[n-k], k > 0$  is  $\frac{z^{-k}}{1 - \frac{1}{2}z^{-1}}$  with region of convergence being

a)  $|z| < 2$  b)  $|z| > 2$  c)  $|z| < \frac{1}{2}$  d)  $|z| > \frac{1}{2}$

Q18. Suppose for input  $x(t)$  a linear time-invariant system with impulse response  $h(t)$  produces output  $y(t)$ , so that  $x(t) * h(t) = y(t)$ . Further if  $|x(t)| * |h(t)| = z(t)$ ,

Which of the following statement is true ?

- (A) For all  $t \in (-\infty, \infty)$ ,  $z(t) \leq y(t)$
- (B) For some but not all  $t \in (-\infty, \infty)$ ,  $z(t) \leq y(t)$
- (C) For all  $t \in (-\infty, \infty)$ ,  $z(t) \geq y(t)$
- (D) For some but not all  $t \in (-\infty, \infty)$ ,  $z(t) \geq y(t)$

Q19. The causal realization of a system transfer function  $H(s)$  having poles at  $(2, -1), (-2, 1)$  and zeroes at  $(2, 1), (-2, -2)$  will be

- a) Stable, real, all pass
- b) unstable, complex, all Pass.
- c) unstable, real, highpass
- d) stable, complex, lowpass

Q20 Consider a signal  $x[n] = (1/2)^n 1[n]$ , where  $1[n] = 0$  if  $n < 0$  and  $1[n] = 1$  if  $n \geq 0$ . The Z-transform of  $x[n-k], k > 0$  is  $\frac{z^{-k}}{1 - \frac{1}{2}z^{-1}}$  with region of convergence being

- a)  $|z| < 2$
- b)  $|z| > 2$
- c)  $|z| < 1/2$
- d)  $|z| > 1/2$

Q21. Consider a linear time-invariant system whose input  $r(t)$  and output  $y(t)$  are related by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 4y(t) = 6r(t)$$

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The poles are at

- a)  $+2j, -2j$    b)  $+2, -2$    c)  $+4, -4$    d)  $+4j, -4j$

Year 2019

Q22 Let  $Y(s)$  be the unit -step response of a causal system having transfer function

$$G(s) = \frac{3 - s}{(s + 1)(s + 3)}$$

that is,  $Y(s) = \frac{G(s)}{s}$ . The forced response of the system is

- (A)  $u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$       (B)  $2u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$   
(C)  $2u(t)$       (D)  $u(t)$

Q23 Consider the signal

Where  $t$  is in second .Its fundamental time period, in seconds is

$$f(t) = 1 + 2 \cos(\pi t) + 3 \sin\left(\frac{2\pi}{3}t\right) + 4 \cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right), \text{ where } t \text{ is in}$$

Q24.

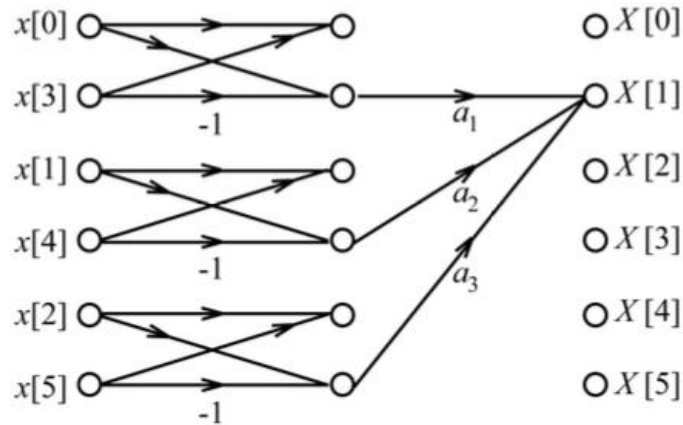
Consider a differentiable function  $f(x)$  on the set of real numbers such that  $f(-1) = 0$  and  $|f'(x)| \leq 2$ . Given these conditions, which one of the following inequalities is necessarily true for all  $x \in [-2, 2]$  ?

- (A)  $f(x) \leq \frac{1}{2}|x + 1|$       (B)  $f(x) \leq 2|x + 1|$   
(C)  $f(x) \leq \frac{1}{2}|x|$       (D)  $f(x) \leq 2|x|$

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Q25.

Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to  $X[1]$  is shown in the figure. Let  $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$ . In the figure, what should be the values of the coefficients  $a_1, a_2, a_3$  in terms of  $W_6$  so that  $X[1]$  is obtained correctly?



- (A)  $a_1 = -1, a_2 = W_6, a_3 = W_6^2$       (B)  $a_1 = 1, a_2 = W_6^2, a_3 = W_6$   
 (C)  $a_1 = 1, a_2 = W_6, a_3 = W_6^2$       (D)  $a_1 = -1, a_2 = W_6^2, a_3 = W_6$

Q26.

The inverse Laplace transform of  $H(s) = \frac{s+3}{s^2+2s+1}$  for  $t \geq 0$  is

- (A)  $3te^{-t} + e^{-t}$       (B)  $3e^{-t}$   
 (C)  $2te^{-t} + e^{-t}$       (D)  $4te^{-t} + e^{-t}$

Q27

The inverse Laplace transform of  $H(s) = \frac{s+3}{s^2+2s+1}$  for  $t \geq 0$  is

- (A)  $3te^{-t} + e^{-t}$       (B)  $3e^{-t}$   
 (C)  $2te^{-t} + e^{-t}$       (D)  $4te^{-t} + e^{-t}$

Q28.

Which one of the following functions is analytic in the region  $|z| \leq 1$ ?

- (A)  $\frac{z^2-1}{z}$       (B)  $\frac{z^2-1}{z+2}$       (C)  $\frac{z^2-1}{z-0.5}$       (D)  $\frac{z^2-1}{z+j0.5}$

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Q29.

The output response of a system is denoted as  $y(t)$ , and its Laplace transform is given by

$$Y(s) = \frac{10}{s(s^2 + s + 100\sqrt{2})}$$

The steady state value of  $y(t)$  is

- (A)  $\frac{1}{10\sqrt{2}}$                       (B)  $10\sqrt{2}$                       (C)  $\frac{1}{100\sqrt{2}}$                       (D)  $100\sqrt{2}$

Q30.

The characteristic equation of a linear time-invariant (LTI) system is given by

$$\Delta(s) = s^4 + 3s^3 + 3s^2 + s + k = 0.$$

The system is BIBO stable if

- (A)  $0 < k < \frac{12}{9}$     (B)  $k > 3$   
 (C)  $0 < k < \frac{8}{9}$     (D)  $k > 6$

Q31

A periodic function  $f(t)$ , with a period of  $2\pi$ , is represented as its Fourier series,

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt.$$

If

$$f(t) = \begin{cases} A \sin t, & 0 \leq t \leq \pi \\ 0, & \pi < t < 2\pi \end{cases},$$

the Fourier series coefficients  $a_1$  and  $b_1$  of  $f(t)$  are

- (A)  $a_1 = \frac{A}{\pi}; b_1 = 0$     (B)  $a_1 = \frac{A}{2}; b_1 = 0$   
 (C)  $a_1 = 0; b_1 = A/\pi$     (D)  $a_1 = 0; b_1 = \frac{A}{2}$

YEAR 2018

Q32 The input be  $u$  and the output be  $y$  of a system, and the other parameters are real constants. Identify which among the following systems is not a linear system:

- (A)  $\frac{d^3y}{dt^3} + a_1 \frac{d^2y}{dt^2} + a_2 \frac{dy}{dt} + a_3 y = b_3 u + b_2 \frac{du}{dt} + b_1 \frac{d^2u}{dt^2}$  (with initial rest conditions)  
 (B)  $y(t) = \int_0^t e^{\alpha(t-\tau)} \beta u(\tau) d\tau$   
 (C)  $y = au + b, \quad b \neq 0$   
 (D)  $y = au$

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Q33. Consider  $(s) = s^3 + a_2s^2 + a_1s + a_0$  with all real coefficients. It is known that its derivative  $p'(s)$  has no real roots. The number of real roots of  $(s)$  is

(A) 0 (B) 1 (C) 2 (D) 3

Q34

Let  $f(x, y) = \frac{ax^2 + by^2}{xy}$ , where  $a$  and  $b$  are constants. If  $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$  at  $x = 1$  and  $y = 2$ , then the relation between  $a$  and  $b$  is

(A)  $a = \frac{b}{4}$                       (B)  $a = \frac{b}{2}$                       (C)  $a = 2b$                       (D)  $a = 4b$

Q35

A discrete-time all-pass system has two of its poles at  $0.25\angle 0^\circ$  and  $2\angle 30^\circ$ . following statements about the system is TRUE?

- (A) It has two more poles at  $0.5\angle 30^\circ$  and  $4\angle 0^\circ$ .
- (B) It is stable only when the impulse response is two-sided.
- (C) It has constant phase response over all frequencies.
- (D) It has constant phase response over the entire  $z$ -plane.

Q36

Let  $x(t)$  be a periodic function with period  $T = 10$ . The Fourier series coefficients for this series are denoted by  $a_k$ , that is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

The same function  $x(t)$  can also be considered as a periodic function with period  $T' = 40$ . Let  $b_k$  be the Fourier series coefficients when period is taken as  $T'$ . If  $\sum_{k=-\infty}^{\infty} |a_k| = 16$ , then  $\sum_{k=-\infty}^{\infty} |b_k|$  is equal to

(A) 256                      (B) 64                      (C) 16                      (D) 4

Q37.

The position of a particle  $y(t)$  is described by the differential equation:

$$\frac{d^2y}{dt^2} = -\frac{dy}{dt} - \frac{5y}{4}.$$

The initial conditions are  $y(0) = 1$  and  $\left. \frac{dy}{dt} \right|_{t=0} = 0$ . The position (accurate to two decimal places) of the particle at  $t = \pi$  is \_\_\_\_\_.

Q38.

Let  $X[k] = k + 1, 0 \leq k \leq 7$  be 8-point DFT of a sequence  $x[n]$ ,

where  $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}$ .

The value (correct to two decimal places) of  $\sum_{n=0}^3 x[2n]$  is \_\_\_\_\_.

Q39. The three roots of the equation  $(x) = 0$  are  $x = \{-2, 0, 3\}$ . What are the three values of  $x$  for which  $(x - 3) = 0$ ?

1. (A) -5,-3,0      (B) -2,0,3      (C) 0,6,8      (D) 1,3,6

Q40

For what values of  $k$  given below is  $\frac{(k+2)^2}{k-3}$  an integer?

(A) 4, 8, 18

(B) 4, 10, 16

(C) 4, 8, 28

(D) 8, 26, 28

Q41

Consider a system governed by the following equations

$$\frac{dx_1(t)}{dt} = x_2(t) - x_1(t)$$

$$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t)$$

The initial conditions are such that  $x_1(0) < x_2(0) < \infty$ . Let  $x_{1f} = \lim_{t \rightarrow \infty} x_1(t)$  and  $x_{2f} = \lim_{t \rightarrow \infty} x_2(t)$ . Which one of the following is true?

- (A)  $x_{1f} < x_{2f} < \infty$     (B)  $x_{2f} < x_{1f} < \infty$     (C)  $x_{1f} = x_{2f} < \infty$     (D)  $x_{1f} = x_{2f} = \infty$

Q42. The number of roots of the polynomial,  $s^7 + 7s^6 + 14s^5 + 31s^4 + 73s^3 + 25s + 200$ , in the open left half of the complex plane is

- A) 3 B) 4 C) 5 D) 6



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Q43.

If  $C$  is a circle  $|z| = 4$  and  $f(z) = \frac{z^4}{(z^2 - 3z + 2)^2}$ , then  $\oint_C f(z) dz$  is

- (A) 1                      (B) 0                      (C) -1                      (D) -2

Q44.

Consider the two continuous-time signals defined below:

$$x_1(t) = \begin{cases} |t|, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}, \quad x_2(t) = \begin{cases} 1 - |t|, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

These signals are sampled with a sampling period of  $T = 0.25$  seconds to obtain discrete-time signals  $x_1[n]$  and  $x_2[n]$ , respectively. Which one of the following statements is true?

- (A) The energy of  $x_1[n]$  is greater than the energy of  $x_2[n]$ .  
 (B) The energy of  $x_2[n]$  is greater than the energy of  $x_1[n]$ .  
 (C)  $x_1[n]$  and  $x_2[n]$  have equal energies.  
 (D) Neither  $x_1[n]$  nor  $x_2[n]$  is a finite-energy signal.

Q45 The signal energy of the continuous-time signal  $x(t) = [(t-1)u(t-1)] - [(t-2)u(t-2)] - [(t-3)u(t-3)] + [(t-4)u(t-4)]$  is

- (A) 11/3 (B) 7/3 (C) 1/3 (D) 5/3

Q46.

The Fourier transform of a continuous-time signal  $x(t)$  is given by

$$X(\omega) = \frac{1}{(10 + j\omega)^2}, \quad -\infty < \omega < \infty, \quad \text{where } j = \sqrt{-1} \text{ and } \omega \text{ denotes frequency. Then the}$$

value of  $|\ln x(t)|$  at  $t = 1$  is \_\_\_\_\_ (up to 1 decimal place). (ln denotes the logarithm to base  $e$ )